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STOCHASTIC MODEL IN FATIGUE CRACK GROWTH

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ABSTRACT

Stochastic modeling of any real time situation falls in four major categories(i)discrete in observation taking values in discrete space (ii) discrete in observation taking values in continuous space (iii) continuous in observation taking values in discrete space (iv) continuous in observation taking values in continuous. We list the fatigue crack growth in the important consideration in structural design and for continued operation of safety critical structural components. We study the major types of probabilistic modeling of fatigue crack growth in any rigid body in that structural material. We substantiate suitability and sustainability of the best type in fatigue crack growth at structural materials.

Keywords: *Fatigue crack growth, Paris Law, Markov chain model, Paris Erdogan model.*

I. INTRODUCTION

We give the various models for random fatigue crack growth. We give an elaborate account of the work by Sobczyk. Crack growth model under random loading is dealt at length .Crack growth is given as a differential equation model and as a Markov chain model .Crack propagation is illustrated as a diffusion model .We have found that stochastic modeling given by Keith Ortiz is best suited for fatigue crack growth.

Traffic growth has always been much greater than predicted and with the development of new materials whose properties cannot be obtained using empirical methods, the need to predict the remaining lives of pavements and the design of new

Pavements to with stand heavier traffic loading with new axle and suspension configurations require the use of an analytical method as the traditional empirical ones cannot cope.

It is desirable to formulate probabilistic models for fatigue phenomena that deal with all physical and chemical processes within a considered material which we observe are likely to be responsible for the generation of fatigue. While the existing physical theories are helpful in explaining qualitatively, the nature of fatigue, they do not give a base for treating fatigue problems quantitatively to yield results that are valid at the macroscopic level. In modeling fatigue processes, it seems to be important to relate the random factors and processes provoking fatigue, the mechanism of fatigue crack growth. It deals with the randomization methodology of investigation of fatigue crack growth through various models. It considers the Paris law with modified versions and discusses the fatigue failure or equivalently the propogation of cracks under random loading. The standard Paris Erdogen model is considered and some of the parameters in the governing equation are randomized for studying crack growth mechanism of materials.

Probabilistic damage models based on Markov chain theory is studied. A basic concept in the model is a duty cycle which is repetitive period of operation on the life of a component during which damage accumulates in a probabilistic manner only on the duty cycle itself and on the value of the damage accumulated at the start of the duty cycle.

On considered the growth of fatigue crack as a continuous stochastic process and obtained it as a diffusion process. Diffusion model of crack propagation is studied. It deals with the integrated of the two probabilistic approaches involving randomization of fatigue growth law co-efficient.

II. STOCHASTIC CRACK GROWTH EQUATION

We improve the model for random fatigue by accounting for randomness of other factors provoking fatigue.

The model proposed is

$$\frac{dL}{dt} = C\mu(t)g(Q)(K_{rms})^n X(t,\gamma) \ \dots\dots\dots\ (1)$$

$$L(t_o) = L_o$$

Here $X(t,\gamma)$ represents the combined effect of unknown random factors of external origin such as environment, creep etc.,

If $X(t,\gamma) = m_x + \overline{X}(t,\gamma)$ where M_x is the average value of $X(t,\gamma)$

$$\frac{dL}{dt} = C\mu(t)g(Q)(K_{rms})^n[m_x + \overline{X}(t,\gamma)] \ \dots\dots\ (2)$$

The above equation can be rewritten as

$$\frac{dL}{dt} = C\mu(t)m_x g(Q)(K_{rms})^n + C\mu(t)g(Q)(K_{rms})^n \overline{X}(t,\gamma) \dots\dots\ (3)$$

It is convenient to write the stochastic growth model (2),(3) in the form

$$\frac{dL}{dt} = a(L,t) + \sigma(L.t)\overline{X}(t,\gamma) \ \dots\dots\dots\ (4)$$

$$L(t_o) = L_o$$

Or

$$\frac{dL}{dt} = f_1(t)L^p(t) + f_2(t)L^p(t)\,\overline{X}(t,\gamma) \ \dots\dots\dots\dots\ (5)$$

$$L(t_o) = L_o$$

Where $p = n/2, \quad C = c\pi^p$

$a(L,t) = f_1(t)L^p(t)$

$\sigma(L,t) = f_2(t)L^p(t)$

$$f_1(t) = m_x\, C\mu(t)g(Q){S_{rms}}^{2p}(t) \ \dots\dots\ (6)$$

$$f_2(t) = 1/m_x f_1(t)$$

There are three basic factors which determine properties and admissibility of the model proposed. They are

i. Random applied stress $S(t,\gamma)$.If it is stationary

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$$S_{rms}(t) = S_o = Constant$$
$$Q(t) = Q_o = Constant$$
$$\mu(t) = \mu_o$$
$$f_1(t) = {f_1}^0 = Constant$$
$$f_2(t) = {f_2}^0 = Constant$$

ii. Random multiplicative noise $X(t,\gamma)$
iii. Constant experimental parameters especially realistic values of 'p'.

A. Properties of Stochastic growth model

Since stochastic differential equation (1) and (2) has been introduced in a somewhat artificial way (via) randomization of experimental laws one should check carefully its properties and usefulness. In order to recognize the basic properties of the model introduced we consider a special but important case when random disturbance $X(t,\gamma)$ is a white Gaussian noise, It is worth nothing that because of normality of the distribution it can theoretically take negative values with positive probability. This deficiency is not serious since the deterministic term in (4) which is positive dominates the tendency of the motion.

Let us assume that in equations (3) and (4)

$$\overline{X}\,(t,\gamma) = \xi(t,\gamma)\ldots\ldots..\ (7)$$

$$< \xi(t_1,\gamma)\xi(t_2,\gamma) > = 2D\delta(t_2 - t_1)$$

Where D is a constant intensity of noise and <. > denotes averaging. Although (4) together with $X(t,\gamma)$ given by (7) looks like a differential equation, it is really G formal record of symbols since $\xi(t,\gamma)$ is an abstraction and not a physical process. Equation (5) with (7) does not define a stochastic process $L(t,\gamma)$ yet, it is a pre-equation.

There are two well known interpretations of our pre-equation turning it into a meaningful stochastic differential equation defining process $L(t,\gamma)$ .These are the Ito and Stratonovich interpretation. Here we adopt Stratowcich interpretation .This means that Ito equation can be understood as the following equivalent Ito Stratonovich interpretation,

$$dL(t) = a^*(L,t)dt + \sigma(L,t)dw(t)\ldots\ldots\ (8)$$

$$L(t_o) = L_o$$

Where w(t) is the Brownian motion starting from $t = t_o$ and $w(t_o) = 0$ almost surely white noise $\xi(t)$ is a generalized derivative of w(t) and

$$a^*(L,t) = a(L,t) + D\sigma(L,t)\,\partial\sigma/\partial L$$

$$= f_1(t)L^p(t) + pD{f_2}^2(t)L^{2p-1}(t)\ldots\ldots\ (9)$$

It should be noted that in the Stratonovich interpretation the deterministic (drift) term $a^*(L,t)$ differs from the macroscopic deterministic law equations (14) and (15).

B. Case 1

Adopting the results of general theory of stochastic systems, analysis of equation (29) yields the following conclusions.

1. In the case when occurring in experimental laws is not greater than one the crack growth is stable in a sense that on each finite time interval L(t) takes finite values with probability one.

   When p=1, what is often met in experimental predictions, the crack size L(t) is expressed explicitly by qualities occurring in the problem i.e. by characteristics of random applied stress, the Brownian motion process W(t) characterizing other uncertainties and initial crack length L_o. When random applied stress is stationary the appropriate formulae are very simple. The probability distribution of the crack size is for each fixed t and deterministic L_o log-normal.
2. In the case where large variety of experimental laws, situation predicted by the model differs from this when ≤ 1.
3. The parameters occurring in the model proposed which need to be estimated from experiments are the following:
   Two material constants:
   C and $p = \frac{n}{2}$ occurring in crack growth law.

   One Constant: D characterizing the intensity of white noise ξ(t).

   Three functions: $m_s(t) =< S(t,\gamma).S_{rms}(t)$And $\mu(t)$;
   If random applied stress is stationary then: $m_s(t) = m_s = constant.$ $S_{rms}(t) = f_o = constant$ and $\mu(t) = \mu_0 = constant.$

III. A DIFFERENTIAL EQUATION MODEL FOR RANDOM FATIGUE GROWTH

Stochastic models for cumulative damage describe the probabilistic mechanism of fatigue accumulation from which fatigue life can be predicted. These models can be broadly classified into two categories. (a) Time to failure models where the system is characterized through a distribution function for the time to failure. (b) 'Phenomenological' models which characterize explicitly the underlying physical mechanism which causes the failure. Cumulative damage models are phenomenological models in which shock causes a certain amount of damage is additive. System failure occurs when total damage exceeds a critical level.

Most of our basic knowledge on fatigue behavior comes from experiments. The experimental data from engineering laboratories constitute a basic source of information about the fatigue behavior of materials subjected to various loading conditions. An important problem is to represent the information contained in the fatigue data in the language of Mathematics.

In the study of crack propogation in materials, most of the researchers take into consideration the paris-Erdogan equation.

$$\frac{da}{dN} = C(\Delta K)^m$$

For the rate of fatigue growth under homogeneous cyclic stressing which was attained from experimental results as the linear regression of on where 'a' is the crack length. 'N' is the number of cycles and k is the range of stress intensity factor at the crack tip. For the rate of fatigue growth under homogeneous cyclic stressing which was attained from experimental results as the linear regression of $log\left[\frac{da}{dN}\right]$ on $log(\Delta K)$ where ‘a’ is the crack length. ‘N’ is the number of cycles and k is the range of stress intensity factor at the crack tip.

In experimental investigations of fatigue crack growth under constant amplitude cyclic loading one finds that curves relating fatigue crack length ‘a’ to cycle number ‘n exhibit certain regularities though they may vary from specimen to specimen. Efforts were made in designing the method of imbedding a probabilistic structure upon such a collection on experimentally obtained curves that could be of use in applications. One such approach to introducing a probabilistic structure is to start with some deterministic differential equation description and introduce random variables in place of the parameters. The Paris-Erdogan model is one among the large number of extensively used model for studying crack growth mechanism in materials. Here we consider various forms of randomization of Paris equation, the properties of its stochastic process solutions as models of real crack growth data. This approach was dealt with by Kozin and Bogdanoff(9)

A. The Paris-Erdogan equation

We consider the paris-Erdogan model from two points of view namely.

Crack length ‘a’ is the dependent variable.

Cycle number ‘n’ is the dependent variable.

It is generally accepted that fatigue crack under constant amplitude cyclic loading can be related to the stress intensity factor Δk through the first order differential equation.

$$\frac{da}{dN} = C(\Delta K)^m \text{.......... (10)}$$

Where $a = a(n)$ the crack length at time n.In general ‘f’ is is an experimentally determined function and ‘c’ is an experimentally determined constant. Clearly $\frac{da}{dn} \geq 0$ implies that c and f are non –ve.From fracture mechanics, one can relate Δk to crack length ‘a’ via

$$\Delta k = \alpha \Delta S a a^{1/2} \text{ .... (11)}$$

Where ‘a’ is a geometrically related parameter and Δs is the applied stress amplitude. (10) can be written as

$$\frac{da}{dn} = cf\left(\alpha \Delta S a^{1/2}\right) \text{ .... (12)}$$

On the basis of number of experimental investigations ‘f’ can be approximated as a power function so that

$$\frac{da}{dn} = c(a\Delta s)^{\beta} a^{\beta} = Ka^{\beta/2} \text{....... (13)}$$

Where k contains all geometric, stress and material factors or parameters. The above equation (13) contains three parameters k, β and a .Thus any randomization of the P.E. equation must be based upon a randomization of these three parameters.

$$\text{Now } \int \frac{da}{a^{\beta/2}} = \int kdn$$

$$\text{i.e. } \frac{a^{-\beta/2}+1}{-\beta/2+1} = kn + C$$

We have the initial condition $a(0) = a_o$.

$$\frac{2}{2-\beta}\, {a_o}^{2-\beta/2} = C$$

$$\frac{2-\beta}{2}\, a^{2-\beta/2} = kn + \frac{2}{2-\beta}\, {a_o}^{2-\beta/2}$$

$$a(n) = {a_o}^{2-\beta/2} + \tfrac{2-\beta}{2} kn^{2/(2-\beta)} = k(a_o) \ldots\ldots (14)$$

IV. CONCLUSION

The conclusion to be drawn regarding stochastic modeling is that

I. For very short crack length auto correlation function must be known.
II. For medium crack lengths, the noise can be treated as uncorrelated white noise.
III. For very long cracks only the random co-efficient of the growth law are necessary.

Thus uncertainty of fatigue life predictions in most design problems, the effects of material inhomogenities are not negligible. In these situations, the researcher must first acknowledge the existence of the randomness before declaring it negligible.

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